CPS4: Dynamical Systems
Credits: 3
Course number: (Special Topics 6501/department equivalent)
Instructor: Dan Quinn

Description
This is a core Cyber Physical Systems (CPS) class. The course will focus on topics that are critical to CPS applications, e.g. pipe networks, lift/drag on vehicles, and relative reference frames in the context of autonomous vehicles and satellites, while avoiding topics that are specific only to advanced Mechanical Engineering, e.g. fluid instabilities and complex potential functions. The course objectives include:
- produce and solve equations of motion for rigid bodies, systems of rigid bodies, elastic solids, and fluid flows
- simulate body dynamics for use in model-based control
- differentiate between solid/fluid behavior over a range of scales (e.g. blood vessels to Micro Aerial Vehicles to atmospheric currents)
- model heat and work fluxes and understand how they factor into the energy budgets of dynamical systems
- think critically about fluid flows and gather physical insights from pressure and velocity fields
- model solid/fluid dynamics of pipe and duct flows, flows through turbines and rotors, flows over wings and projectiles

Course Outline
Introduction to Solid Dynamics (Classes 1-2)
- Free-body diagrams
- Cross product and moments
- Equations of equilibrium
- Reduction of distributed loadings

Structural analysis (Classes 3-5)
- Trusses and frames
- Support and connection types
- The Method of Sections
- Sample CPS Application: Sensor networks to evaluate loads on bridges

Friction (Classes 6-8)
- Static and kinetic friction
- Rolling resistance
- Bearings
- Sample CPS Application: Stopping distance in autonomous vehicles

Kinematics of rigid bodies (Classes 9-11)
- Moment of inertia
- Equations of motion: translation and rotation
- Impulse and momentum
- Relative reference frames
- Sample CPS Application: Model-based control and motion-planning

Work and energy (Classes 12-14)
- Kinetic and potential energy
- Conservative and non-conservative forces
- Damped harmonic oscillators
- Sample CPS Application: Vibration harvesting in wearable bio-sensors

Introduction to fluid dynamics (Classes 15-16)
- Pressure, density, temperature
- Control volume analysis
- Navier-Stokes Equation and classic assumptions
- Inviscid vs. viscous flows
  - Reynolds number
  - Bernoulli Equation + viscous corrections
- Turbulence
- Computational Fluid Dynamics

Fluid flow in infrastructure (Classes 17-19)
- Steady pipe flows, e.g. oil pipelines
- Laminar vs. turbulent pipe flow
- Incompressible flow networks, e.g. plumbing, HVAC
- Fans and pumps
- Turbines, e.g. wind farms
- Sample CPS Application: Temperature management in HVAC

Fluid flow in the body (Classes 20-22)
- Pulsatile pipe flows, e.g. windpipes, arteries
- Turbulent jets
- Air, blood, urine flows
- Inflammation, blockage, and stents
- Sample CPS Application: Implantable body flow sensors

Fluid flow over vehicles (Classes 23-25)
- No slip condition and boundary layers
- Drag on streamlined and bluff bodies
- Wing lift and stall
- Disk actuator theory & propeller design
- Sample CPS Application: Model-based control in quadrotor UAVs
Sample Lesson Plan

**Class 11: Sample CPS Application: Model-based control and motion-planning**

*Class objectives:* Students will learn...
- how conservation laws can model the motion of an inverted pendulum
- how a data-driven controller could be used to hold up an inverted pendulum
- ways in which a data-driven controller could fail to hold up a pendulum
- dimensionless parameters that govern the conditions where a controller fails
- ways that model-based control could improve the pendulum controller

*Before Class:* As homework due before this class, pairs of students will have prepared MATLAB/Python code that solves for the motions of a simple beam given a time history of forces and moments on the beam.

**Activity 1: Introducing inverted pendulum physics (~10 min)**
The instructor will show students how a point load applied at the base of a beam can model an inverted pendulum being held up from its base. A volunteer will be asked to balance a yard stick for the class to appreciate some of the control challenges of an inverted pendulum.

**Activity 2: Exploring P-control of the pendulum (~25 min)**
Students will work in pairs (the same pairs that prepared the homework) to simulate a pendulum controlled by a P-controller (point force angle will be a constant gain times pendulum angle with constant latency). Students will discover conditions where the controller fails.

**Activity 3: Exploring limitations of data-driven control (~15 min)**
The instructor will show a simulation of a P controller holding up a pendulum. Based on their exploration in pairs, students will suggest conditions where the controller may fail (short, heavy pendulum; high latency) until all students have seen multiple failure types. The instructor will derive dimensionless parameters that combine physical variables (mass, length, etc.) with cyber variables (latency, gain) to prove certain ratios of gain/latency/mass/length are doomed to fail.

**Activity 4: Discuss ways physical models could improve control (~15 min)**
The instructor will introduce more advanced data-driven approaches (PID and beyond), but show there are still limitations. Students will brainstorm ways that momentum conservation laws could be used to create model-based control.